Calculators, Mobile Phones and Pagers are not allowed

Answer all of the following questions

1. Use linear approximation to estimate $f(7.1)$ where $f(x) = 3(1+x)^{\frac{1}{3}}$ (3 pts.)

2. If $f(x) = 1 + 2(x-8)^{\frac{2}{3}}$, show that $f(0) = f(16)$ but $f'(c) \neq 0$ for every number c in the open interval $(0, 16)$ Why doesn't this contradict Rolle's theorem? (3 pts.)

3. Show that $f(x) = 1 + x - x^2 - x^4$ has no local minimum? (3 pts.)

4. Find an equation of the normal line to the curve $x^3 + 2x^2y + y^2 = 1$ at the point $P(1, 0)$ (4 pts.)

5. A cone of metal whose altitude is twice its base radius, is heated so that its base radius increases at a rate of 0.01 cm/min. Find the rate at which its volume is changing when the base radius is 10 cm. (4 pts.)

6. If $f(x) = 2x^3 - 6x + 11$, find the maximum and minimum values of f on the interval $[0, 2]$. (4 pts.)

7. Let f be a differentiable function such that:

   (a) $f(-1) = 0, f(0) = 3, f(1) = 2$ and $f(3) = 1$

   (b) $f'(x) > 0$ on $(-\infty, 0) \cup (3, \infty)$ and $f'(x) < 0$ on $(0, 3)$.

   (c) $f''(x) < 0$ on $(-\infty, 1)$ and $f''(x) > 0$ on $(1, \infty)$.

   Sketch the graph of $y = f(x)$, showing the local extrema and the points of inflection if any. (4 pts.)